MATHEMATICS

Chapter 14: Probability

Probability

1. **Probability** is a quantitative measure of uncertainty.
2. In the **experimental approach** to probability, we find the probability of the occurrence of an event by actually performing the experiment a number of times and adequate recording of the happening of event.
3. In the **theoretical approach** to probability, we try to predict what will happen without actually performing the experiment.
4. The experimental probability of an event approaches to its theoretical probability if the number of trials of an experiment is very large.
5. An **outcome** is a result of a single trial of an experiment.
6. The word **'experiment'** means an operation which can produce some well defined outcome(s). There are two types of experiments:
   i. **Deterministic experiments:** Experiments which are repeated under identical conditions produce the same results or outcomes are called deterministic experiments.
   ii. **Random or Probabilistic experiment**: If an experiment, when repeated under identical conditions, do not produce the same outcome every time but the outcome in a trial is one of the several possible outcomes, then it is known as a random or probabilistic experiment.

   In this chapter, the term experiment will stand for random experiment.
7. The collection of all possible outcomes is called the **sample space**.
8. An outcome of a random experiment is called an **elementary event**.
9. An event associated to a random experiment is a **compound event** if it is obtained by combining two or more elementary events associated to the random experiment.
10. An event *A* associated to a random experiment is said to occur if any one of the elementary events associated to the event *A* is an outcome.
11. An elementary event is said to be **favorable** to a compound event *A*, if it satisfies the definition of the compound event *A*. In other words, an elementary event *E* is favorable to a compound event *A*, if we say that the event *A* occurs when *E* is an outcome of a trial.
12. In an experiment, if two or more events have equal chances to occur or have equal probabilities, then they are called **equally likely events**.
13. The **theoretical probability (also called classical probability) of an event *E***, written as *P* (*E*), is defined as

$$\frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}$$

14. For two events A and B of an experiment:

If $P(A) > P(B)$ then event A is more likely to occur than event B.

If $P(A) = P(B)$ then events A and B are equally likely to occur.

15. An event is said to be **sure event** if it always occur whenever the experiment is performed. The probability of sure event is always one. In case of sure event elements are same as the sample space.

16. An event is said to be **impossible event** if it never occur whenever the experiment is performed. The probability of an impossible event is always zero. Also, the number of favorable outcome is zero for an impossible event.

17. Probability of an event lies between 0 and 1, both inclusive, i.e., $\mathbf{0 \le P(A) \le 1}$

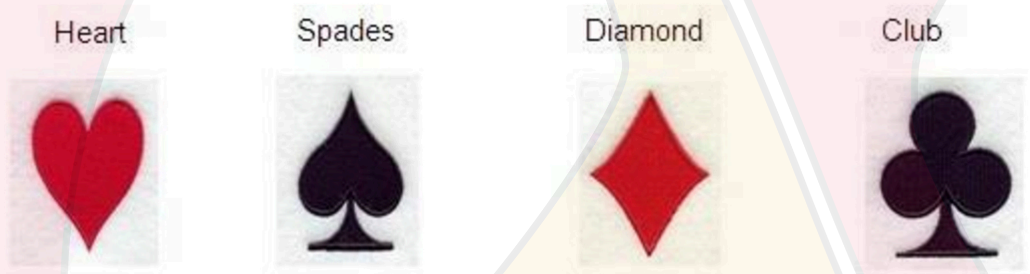
18. If E is an event in a random experiment then the event 'not *E*' (denoted by $\overline{E}$) is called the **complementary event** corresponding to *E*.

19. The **sum of the probabilities** of all elementary events of an experiment is 1.

20. For an event *E*, $\mathbf{P(\overline{E}) = 1 - P(E)}$, where the event $\overline{E}$ representing 'not *E*" is the complement of event E.

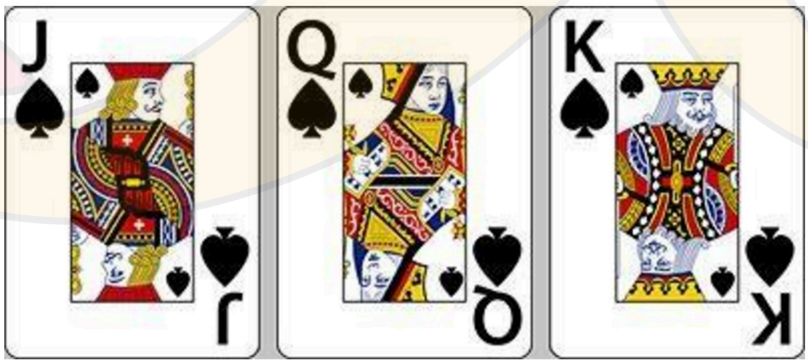
21. Suits of Playing Card

A pack of playing cards consist of 52 cards which are divided into 4 suits of 13 cards each. Each suit consists of one ace, one king, one queen, one jack and 9 other cards numbered from 2 to 10. Four suits are named as spades, hearts, diamonds and clubs.



22. Face Cards

King, queen and jack are face cards.



The formula for finding the **geometric probability** of an event is given by:

$$P(E) = \frac{\text{Measure of the specified part of the region}}{\text{Measure of the whole region}}$$

Here, 'measure' may denote length, area or volume of the region or space.

Event and outcome

An Outcome is a result of a random experiment. For example, when we roll a dice getting six is an outcome.

An Event is a set of outcomes. For example when we roll dice the probability of getting a number less than five is an event.

Note: An Event can have a single outcome

Events and Types of Events in Probability

What are Events in Probability?

A probability event can be defined as a set of outcomes of an experiment. In other words, an event in probability is the subset of the respective sample space. So, what is sample space?

The entire possible set of outcomes of a random experiment is the sample space or the individual space of that experiment. The likelihood of occurrence of an event is known as probability. The probability of occurrence of any event lies between 0 and 1.

The sample space for the tossing of three coins simultaneously is given by:

S = {(T , T , T) , (T , T , H) , (T , H , T) , (T , H , H) , (H , T , T) , (H , T , H) , (H , H, T) ,(H , H , H)}

Suppose, if we want to find only the outcomes which have at least two heads; then the set of all such possibilities can be given as:

E = { (H , T , H) , (H , H ,T) , (H , H ,H) , (T , H , H)}

Thus, an event is a subset of the sample space, i.e., E is a subset of S.

There could be a lot of events associated with a given sample space. For any event to occur, the outcome of the experiment must be an element of the set of event E.

What is the Probability of Occurrence of an Event?

The number of favourable outcomes to the total number of outcomes is defined as the probability of occurrence of any event. So, the probability that an event will occur is given as:

Types of Events in Probability:

Some of the important probability events are:

- Impossible and Sure Events
- Simple Events
- Compound Events
- Independent and Dependent Events
- Mutually Exclusive Events
- Exhaustive Events
- Complementary Events

- Events Associated with "OR"
- Events Associated with "AND"
- Event E1 but not E2

Impossible and Sure Events

If the probability of occurrence of an event is 0, such an event is called an impossible event and if the probability of occurrence of an event is 1, it is called a sure event. In other words, the empty set ϕ is an impossible event and the sample space S is a sure event.

Simple Events

Any event consisting of a single point of the sample space is known as a simple event in probability. For example, if S = {56, 78 , 96 , 54 , 89} and E = {78} then E is a simple event.

Compound Events

Contrary to the simple event, if any event consists of more than one single point of the sample space then such an event is called a compound event. Considering the same example again, if S = {56 ,78 ,96 ,54 ,89}, E1 = {56 ,54 }, E2 = {78 ,56 ,89 } then, E_1 and E_2 represent two compound events.

Independent Events and Dependent Events

If the occurrence of any event is completely unaffected by the occurrence of any other event, such events are known as an independent event in probability and the events which are affected by other events are known as dependent events.

Mutually Exclusive Events

If the occurrence of one event excludes the occurrence of another event, such events are mutually exclusive events i.e. two events don't have any common point. For example, if S = {1, 2 , 3 , 4 , 5 , 6} and E1, E2 are two events such that E1 consists of numbers less than 3 and E2 consists of numbers greater than 4.

So, E1 = {1,2} and E2 = {5,6} .

Then, E1 and E2 are mutually exclusive.

Exhaustive Events

A set of events is called exhaustive if all the events together consume the entire sample space.

Complementary Events

For any event E1 there exists another event E1' which represents the remaining elements of the sample space S.

E1 = S − E1'

If a dice is rolled then the sample space S is given as S = {1 , 2 , 3 , 4 , 5 , 6 }. If event E1 represents all the outcomes which is greater than 4, then E1 = {5, 6} and E1' = {1, 2, 3, 4}.

Thus E1ʹ is the complement of the event E1.

Similarly, the complement of E_1, E_2, E_3.........E_n will be represented as E_1', E_2', E_3'.........E_n'

Experimental Probability

Experimental probability can be applied to any event associated with an experiment that is repeated a large number of times.

A trial is when the experiment is performed once. It is also known as empirical probability.

Experimental or empirical probability: P(E) =Number of trials where the event occurred/Total Number of Trials

You and your 3 friends are playing a board game. It's your turn to roll the die and to win the game you need a 5 on the dice. Now, is it possible that upon rolling the die you will get an exact 5? No, it is a matter of chance. We face multiple situations in real life where we have to take a chance or risk. Based on certain conditions, the chance of occurrence of a certain event can be easily predicted. In our day to day life, we are more familiar with the word 'chance and probability'. In simple words, the chance of occurrence of a particular event is what we study in probability. In this article, we are going to discuss one of the types of probability called "Experimental Probability" in detail.

Theoretical Probability

Theoretical Probability, P(E) = Number of Outcomes Favourable to E / Number of all possible outcomes of the experiment

Here we assume that the outcomes of the experiment are equally likely.

Every one of us would have encountered multiple situations in life where we had to take a chance or risk. Depending on the situation, it can be predicted up to a certain extent if a particular event is going to take place or not. This chance of occurrence of a particular event is what we study in probability. In our everyday life, we are more accustomed to the word 'chance' as compared to the word 'probability'. Since Mathematics is all about quantifying things, the theory of probability basically quantifies these chances of occurrence or non-occurrence of certain events. In this article, we are going to discuss what is probability and its two different types of approaches with examples.

In Mathematics, the probability is a branch that deals with the likelihood of the occurrences of the given event. The probability value is expressed between the range of numbers from 0 to 1. The three basic rules connected with the probability are addition, multiplication, and complement rules.

Theoretical Probability Vs Experimental Probability

Probability theory can be studied using two different approaches:

Theoretical Probability

Experimental Probability

Theoretical Probability Definition

Theoretical probability is the theory behind probability. To find the probability of an event using theoretical probability, it is not required to conduct an experiment. Instead of that, we should know about the situation to find the probability of an event occurring. The theoretical probability is defined as the ratio of the number of favourable outcomes to the number of possible outcomes.

Probability of Event P(E) = No. of. Favourable outcomes/ No. of. Possible outcomes.

Experimental Probability Definition

The experimental probability also is known as an empirical probability, is an approach that relies upon actual experiments and adequate recordings of occurrence of certain events while the theoretical probability attempts to predict what will happen based upon the total number of outcomes possible. The experimental Probability is defined as the ratio of the number of times that event occurs to the total number of trials.

Probability of Event P(E) = No. of. times that event occurs/ Total number of trials

The basic difference between these two approaches is that in the experimental approach; the probability of an event is based on what has actually happened by conducting a series of actual experiments, while in theoretical approach; we attempt to predict what will occur without actually performing the experiments.

Elementary Event

An event having only one outcome of the experiment is called an elementary event.

Example: Take the experiment of tossing a coin n number of times. One trial of this experiment has two possible outcomes: Heads(H) or Tails(T). So for an individual toss, it has only one outcome, i.e Heads or Tails.

Sum of Probabilities

The sum of the probabilities of all the elementary events of an experiment is one.

Example: take the coin-tossing experiment. P(Heads) + P(Tails)

= (1/2)+ (1/2) =1

Impossible event

An event that has no chance of occurring is called an Impossible event, i.e. P(E) = 0.

E.g: Probability of getting a 7 on a roll of a die is 0. As 7 can never be an outcome of this trial.

Sure event

An event that has a 100% probability of occurrence is called a sure event. The probability of occurrence of a sure event is one.

E.g: What is the probability that a number obtained after throwing a die is less than 7?

So, P(E) = P(Getting a number less than 7) = 6/6= 1

Range of Probability of an event

The range of probability of an event lies between 0 and 1 inclusive of 0 and 1, i.e. $0 \le P(E) \le 1$.

Geometric Probability

Geometric probability is the calculation of the likelihood that one will hit a particular area of a figure. It is calculated by dividing the desired area by the total area. In the case of Geometrical probability, there are infinite outcomes.

Complementary Events

Complementary events are two outcomes of an event that are the only two possible outcomes. This is like flipping a coin and getting heads or tails.

$P(E) + P(\overline{E}) = 1$, where E and $\overline{E}$

are complementary events. The event $\overline{E}$, representing 'not E', is called the complement of the event E.

For any event A, there exists another event A' which shows the remaining elements of the sample space S. A' denotes complementary event of A.

A' = S – A.

Event A and A' are mutually exclusive and exhaustive.

Consider the example of tossing a coin. Let P(E) denote the probability of getting a tail when a coin is tossed. Then probability of getting a head is denoted by

$P(\overline{E}). P(E) + P(\overline{E}) = 1$

The event $P(\overline{E})$ means 'Not E'.

Example 1:

A bag contains only lemon-flavoured candies. Arjun takes out one candy without looking into the bag. What is the probability that he takes out an orange-flavoured candy?

Solution:

Let us take the number of candies in the bag to be 100.

Number of orange flavoured candies = 0 [since the bag contains only lemon-flavoured candies]

Hence, the probability that he takes out an orange-flavoured candy is:

P (Taking orange-flavoured candy) = Number of orange flavoured candies / Total number of candies.

= 0/100 = 0

Hence, the probability that Arjun takes out an orange-flavoured candy is 0.

This proves that the probability of an impossible event is 0.

Example 2:

A game of chance consists of spinning an arrow that comes to rest pointing at any one of the numbers such as 1, 2, 3, 4, 5, 6, 7, 8 and these are equally likely outcomes. What is the probability that it will point at (i)8, (ii) Number greater than 2 (iii) Odd numbers.

Solution:

Sample Space = {1, 2, 3, 4, 5, 6, 7, 8}

Total Numbers = 8

(i) Probability that the arrow will point at 8:

Number of times we can get 8 = 1

P (Getting 8) = 1/8.

(ii) Probability that the arrow will point at the number greater than 2:

Number greater than 2 = 3, 4, 5, 6, 7, 8.

No. of numbers greater than 2 = 6

P (Getting numbers greater than 2) = 6/8 = 3/4.

(iii) Probability that the arrow will point at the odd numbers:

Odd number of outcomes = 1, 3, 5, 7

Number of odd numbers = 4.

P (Getting odd numbers) = 4/8 = ½.

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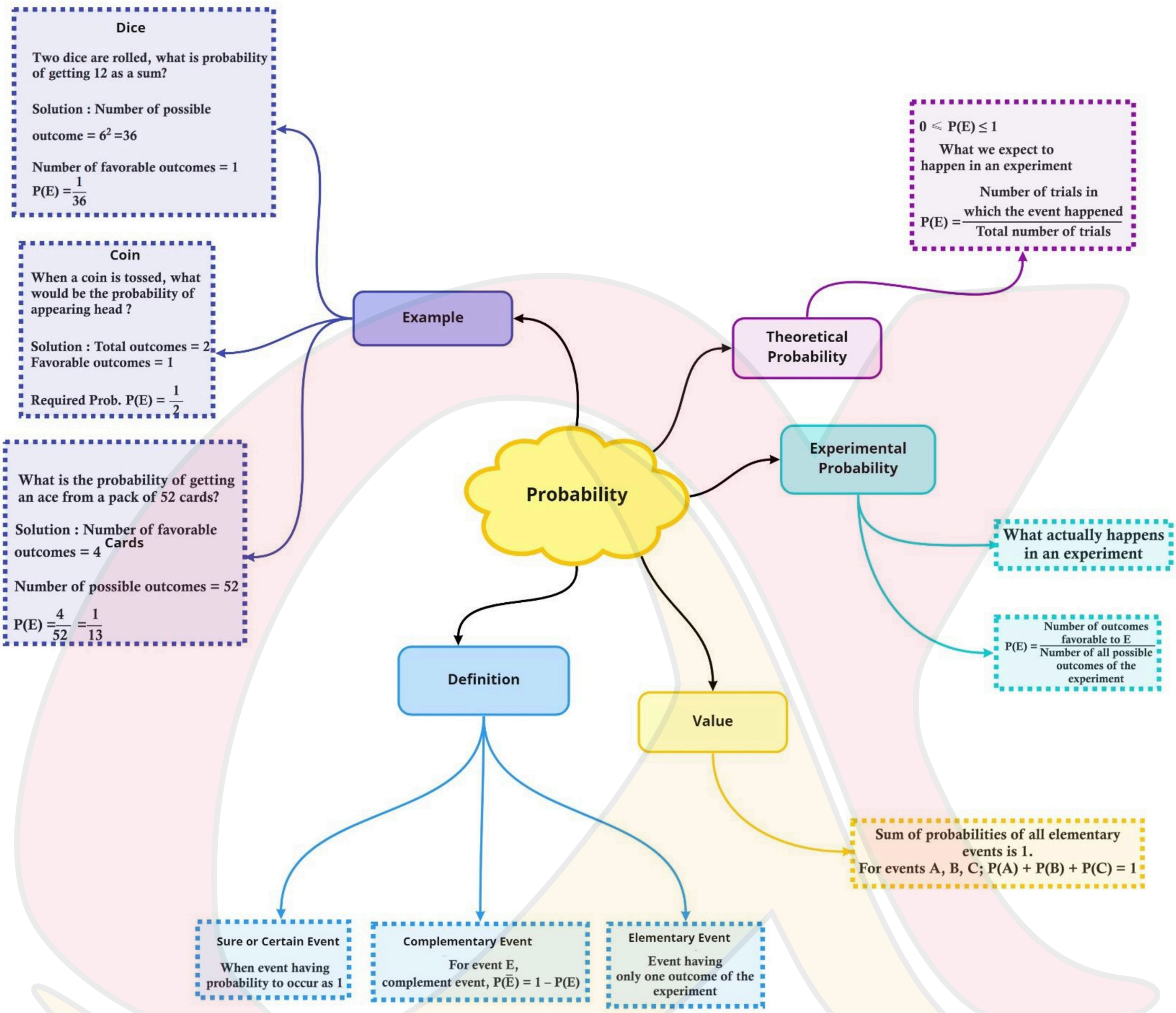


PROBABILITY

Multiple Choice questions-

1. An event is very unlikely to happen. Its probability is closest to:

(a) 0.0001

(b) 0.001

(c) 0.01

(d) 0.1

2. If the probability of an event is P, the probability of its complementary event will be:

(a) P – 1

(b) P

(c) 1 – p

(d) $1 - \frac{1}{p}$

3. If P(A) denotes the probability of an event then:

(a) $P(A) < 0$

(b) $P(A) > 0$

(c) $0 \leq P(A) \leq 1$

(d) $-1 \leq P(A) \leq 0$

4. A card is drawn from a deck of 52 cards. The event E is that card is not an ace of hearts. The number of outcomes favourable to E is:

(a) 4

(b) 13

(c) 48

(d) 51

5. The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is:

(a) 7

(b) 14

(c) 21

(d) 28

6. A girl calculate that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought?

(a) 40

(b) 240

(c) 480

(d) 750

7. One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number which is a multiple of 5 is:

(a) $\frac{1}{5}$
(b) $\frac{3}{5}$
(c) $\frac{4}{5}$
(d) $\frac{1}{3}$

8. Someone is asked to take a number from 1 to 100. The probability that it is a prime is:

(a) $\frac{1}{5}$
(b) $\frac{6}{25}$
(c) $\frac{1}{4}$
(d) $\frac{13}{50}$

9. A school has five houses A, B, C, D and E. A class has 23 students, 4 from house A, 8 from house B, 5 from house C, 2 from house D and rest from house E. A single student is selected at random to be the class monitor. The probability that the selected student is not from A, B and C is:

(a) $\frac{4}{23}$

(b) $\frac{6}{23}$

(c) $\frac{8}{23}$

(d) $\frac{17}{23}$

10. When a die is thrown, the probability of getting an odd number less them 3 is:

(a) $\frac{1}{6}$

(b) $\frac{1}{3}$

(c) $\frac{1}{2}$

(d) 0

Very Short Questions:

1. State true or false and give the reason. If I toss a coin 3 times and get head each tir ne, then I should expect a tail to have a higher chance in the 4th toss.
2. A bag contains slips numbered from 1 to 100. If Fatima chooses a slip at random from the bag, it will either be an odd number or an even number. Since, this situation has only two possible outcomes, so the probability of each is $\frac{1}{2}$ Justify.
3. In a family, having three children, there may be no girl, one girl, two girls or three girls. So, the probability of each is $\frac{1}{4}$. Is this correct? Justify your answer.
4. A game consists of spinning an arrow which comes to rest pointing at one of the regions (1, 2 or 3) Fig. Are the outcomes 1, 2 and 3 equally likely to occur? Give reason.
5. Two coins are tossed simultaneously. Find the probability of getting exactly one head.
6. From a well shuffled pack of cards, a card is drawn at random. Find the probability of getting a black queen.
7. If P (E) = 0.05, what is the probability of 'not E' ?
8. What is the probability of getting no head when two coins are tossed simultaneously?
9. In a single throw of a pair of dice, what is the probability of getting the sum a perfect square?
10. Someone is asked to choose a number from 1 to 100. What is the probability of it being a prime number?

Short Questions :

1. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is a prime number.

2. Find the probability that a number selected from the numbers 1 to 25 is not a prime number when each of the given numbers is equally likely to be selected.

3. One card is drawn at random from a pack of 52 cards. Find the probability that the card drawn is an ace and black.

4. A card is drawn at random from a pack of 52 playing cards. Find the probability that the card drawn is neither an ace nor a king.

5. A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out (i) an orange flavoured candy? (ii) a lemon flavoured candy?

6. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

7. Two players, Sangeeta and Reshma, play a tennis match. It is known that the probability of Sangeeta's winning the match is 0.62. What is the probability of Reshma's winning the match?

8. A child has a die whose six faces show the letters as given below:

   A *B* *C* *D* *E* *A*

   The die is thrown once. What is the probability of getting (i) A? (ii) D?

9. A card is drawn at random from a pack of 52 playing cards. Find the probability that the card drawn is neither a red card nor a black king.

10. Out of 400 bulbs in a box, 15 bulbs are defective. One bulb is taken out at random from the box. Find the probability that the drawn bulb is not defective.

11. Harpreet tosses two different coins simultaneously (say, one is of 1 and other of 2). What is the probability that she gets at least one head?

12. A game consists of tossing a one-rupee coin 3 times and noting the outcome each time. Ramesh wins the game if all the tosses give the same result (i.e. three heads or three tails) and loses otherwise. Find the probability of Ramesh losing the game.

13. Three unbiased coins are tossed together. Find the probability of getting:

(i) all heads.

(ii) exactly two heads.

(iii) exactly one head.

(iv) at least two heads.

(v) at least two tails

14. A die is thrown once. Find the probability of getting:

(i) a prime number.

(ii) a number lying between 2 and 6.

(iii) an odd number

15. Suppose we throw a die once.

(i) What is the probability of getting a number greater than 4?

(ii) What is the probability of getting a number less than or equal to 4?

Long Questions :

1. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting:

(i) a king of red colour.

(ii) a face card.

(iii) a red face card.

(iv) the jack of hearts.

(v) a spade.

(vi) the queen of diamonds.

2. One card is drawn from a pack of 52 cards, each of the 52 cards being equally likely to be drawn. Find the probability that the card drawn is:

(i) an ace.

(ii) red.

(iii) either red or king.

(iv) red and a king.

(v) a face card.

(vi) a red face card.

(vii) “2’ of spades.

(viii) ’10’ of a black suit.

3. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1,2,3,4,5,6,7,8 see Fig, and these are equally likely outcomes. What is the probability that it will point at: (i) 8? (ii) an odd number? (iii) a number greater than 2? (iv) a number less than 9?

4. Two dice, one blue and one grey, are thrown at the same time. Write down all the possible outcomes. What is the probability that the sum of the two numbers appearing on the top of the dice is: (i) 8? (ii) 13? (iii) less than or equal to 12?

5. A bag contains cards numbered from 1 to 49. A card is drawn from the bag at random, after mixing the cards thoroughly. Find the probability that the number on the drawn card is:

(i) an odd number.

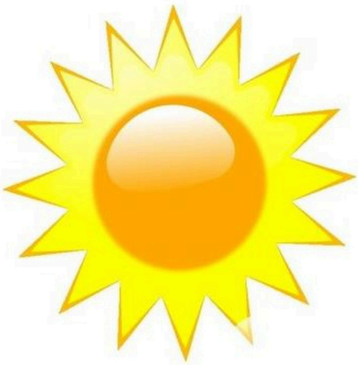
(ii) a multiple of 5.

(iii) a perfect square.

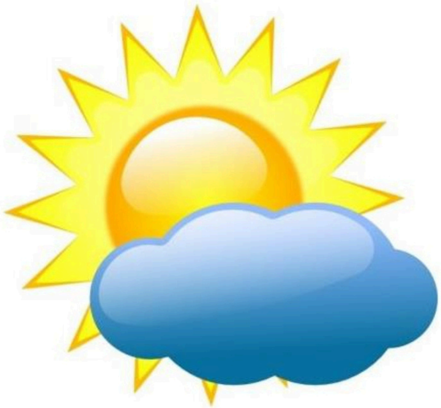
(iv) an even prime number.

Case Study Questions:

1. In the month of May, the weather forecast department gives the prediction of weather for the month of June. The given table shows the probabilities of forecast of different days:



Sunny



Partially cloudy

Cloudy

Rainy

Days	Days	Cloudy	Partially cloudy	Rainy
Probability	$\frac{1}{2}$	x	$\frac{1}{5}$	y

If the forecast is 100% correct for June, then answer the following questions.

i. The number of sunny days in June, is:

a. 5
b. 10
c. 15
d. 20

ii. If the number of cloudy days in June is 5, then x =

a. $\frac{1}{4}$

b. $\frac{1}{6}$

c. $\frac{1}{8}$

d. $\frac{1}{10}$

iii. The probability that the day is not rainy is:

a. $\frac{13}{15}$

b. $\frac{11}{15}$

c. $\frac{1}{15}$

d. None of these

iv. If the sum of x and y is $\frac{3}{10}$, then the number of rainy days in June is:

a. 1
b. 2
c. 3
d. 4

v. Find the number of partially cloudy days.

a. 2
b. 4
c. 6
d. 8

2. Vishal goes to a store to purchase juice cartons for his shop. The store has 80 cartons of orange juice, 90 cartons of apple juice, 38 cartons of mango juice and 42 cartons of guava juice. If Vishal chooses a carton at random, then answer the following questions.

i. The probability that the selected carton is of apple juice is:

a. $\frac{1}{25}$

b. $\frac{8}{25}$

c. $\frac{13}{25}$

d. $\frac{9}{25}$

ii. The probability that the selected carton is not of orange juice is:

a. $\frac{14}{25}$

b. $\frac{11}{25}$

c. $\frac{17}{25}$

d. $\frac{4}{125}$

iii. The probability of selecting a carton of guava juice is:

a. $\frac{51}{125}$

b. $\frac{16}{125}$

c. 0

d. $\frac{21}{125}$

iv. Vishal buys 4 cartons of apple juice, 3 cartons of orange juice and 3 cartons of guava juice. A customer comes to Vishal's shop and picks a tetrapack of juice at random. The probability that the customer picks a guava juice, if each carton has 10 tetrapacks of juice, is:

a. $\frac{1}{10}$

b. $\frac{2}{10}$

c. $\frac{3}{10}$

d. $\frac{2}{5}$

v. If the storekeeper bought 14 more cartons of apple juice, then the probability of selecting a tetrapack of apple juice from the store is:

a. $\frac{25}{127}$

b. $\frac{50}{127}$

c. $\frac{75}{127}$

d. $\frac{100}{127}$

Assertion Reason Questions-

1. ***Directions:*** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

Assertion: The probability of winning a game is 0.4, then the probability of losing it, is 0.6.

Reason: P (E) + P(not E) = 1

2. ***Directions:*** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
   (a) Both A and R are true and R is the correct explanation of A.
   (b) Both A and R are true and R is not the correct explanation of A.
   (c) A is true but R is false.
   (d) A is false but R is true.

Assertion: If the probability of an event is P, then probability of its complementary event will be 1 × P.

Reason: LCM × product of numbers = HCF

Answer Key-

Multiple Choice questions-

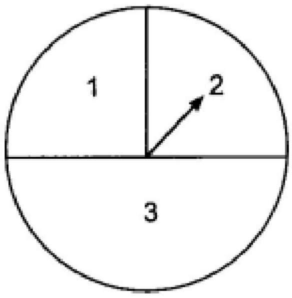
1. (a) 0.0001
2. (c) 1 – p
3. (c) $0 \leq P(A) \leq 1$
4. (d) 51
5. (b) 14
6. (c) 480
7. (a) $\frac{1}{5}$
8. (c) $\frac{1}{4}$
9. (b) $\frac{6}{23}$
10. (a) $\frac{1}{6}$

Very Short Answer :

1. False, because the outcomes 'head' and 'tail are equally likely. So, every time the probability of getting head or tail is $\frac{1}{2}$
2. True, because the outcomes odd number' and `even number' are equally likely here.

3. False, because the outcomes are not equally, likely. For no girl, outcome is bbb, for one girl, it is bgb, gbb, bbg, for two girls, it is bgg, ggb, gbg and for all girls, it is ggg.

4.



False, because the outcome 3 is more likely than the other numbers.

5. Possible outcomes are {HH, HT, TH, TT}.

(exactly one head) $= \frac{2}{4} = \frac{1}{2}$

6. Number of black queens in a pack of cards = 2

$\therefore$ P (black queen) $= \frac{25}{2} = \frac{1}{26}$

7. As we know that,

P (E) + P (not E) = 1

P (not E) = 1 – P (E) = 1 – 0.05 = 0.95

8. Favourable outcome is TT;

$\therefore$ P (no head) $= \frac{1}{4}$

9. Total outcomes = 36

Favourable outcomes are {(1,3), (3, 1), (2, 2), (3, 6), (6,3), (4, 5), (5, 4)}

$\therefore$ Required probability $= \frac{7}{36}$

10. Total prime numbers between 1 to 100 = 25

$\therefore$ P (Prime number) $= \frac{25}{100} = 14$

Short Answer :

1. Product of the number on the dice is prime number, i.e., 2, 3, 5.

The possible ways are, (1, 2), (2, 1), (1, 3), (3, 1), (5, 1), (1, 5)

So, number of possible ways = 6

∴ Required probability $= \frac{6}{36} = \frac{1}{6}$

2. Total prime numbers from 1 to 25 = 9.

∴ Non-prime numbers from 1 to 25 = 25 – 9 = 16.

⇒ P (non-prime number) $= \frac{16}{25}$

3. Number of black aces in a pack of cards = 2

∴ P (an ace and black card) $= \frac{2}{52} = \frac{1}{26}$

4. Let E be the event card drawn is neither an ace nor a king.

Then, the number of outcomes favourable to the event E = 44 (4 kings and 4 aces are not there)

∴ $P(E) = \frac{44}{52} = \frac{11}{13}$

5. (i) As the bag contains only lemon flavoured candies. So, the event related to the experiment of taking out an orange flavoured candy is an impossible event. So, its probability is 0.

(ii) As the bag contains only lemon flavoured candies. So, the event related to the experiment of taking out lemon flavoured candies is certain event. So, its probability is 1.

6. Here, total number of pens = 132 + 12 = 144

∴ Total number of elementary outcomes = 144

Now, favourable number of elementary events = 132

∴ Probability that a pen taken out is good one $= \frac{132}{144} = \frac{11}{12}$

7. Let S and R denote the events that Sangeeta and Reshma wins the match, respectively.

The probability of Sangeeta's winning = P(S) = 0.62

As the events R and S are complementary

∴ The probability of Reshma's winning = P(R) = 1 – P(S)

= 1 – 0.62 = 0.38.

8. The total number of elementary events associated with random experiment of throwing a die is 6.

(i) Let E be the event of getting a letter A.

∴ Favourable number of elementary events = 2

∴ $P(E) = \frac{2}{6} = \frac{1}{3}$

(ii) Let E be the event of getting a letter D.

∴ Favourable number of elementary events = 1

∴ $P(E) = \frac{1}{6}$

9. Let E be the event card drawn is neither a red card nor a black king'

The number of outcomes favourable to the event E = 24 (26 red cards and 2 black kings are not there, so 52 – 28 = 24)

$$\therefore\ P(E) = \frac{24}{52} = \frac{16}{13}$$

10. Total number of bulbs in the box = 400

Total number of defective bulbs in the box = 15

Total number of non-defective bulbs in the box = 400 – 15 = 385

$$P\text{ (bulb is not defective)} = \frac{\text{Number of non - defective bulbs}}{\text{Total number of bulbs}} = \frac{385}{400} = \frac{77}{80}.$$

11. When two coins are tossed simultaneously, the possible outcomes are (H, H), (H, T), (T, H), (T, T) which are all equally likely. Here (H, H) means head up on the first coin (say on ₹ 1) and head up on the second coin (₹ 2). Similarly (H, T) means head up on the first coin and tail up on the second coin and so on.

The outcomes favourable to the event E, 'at least one head' are (H, H), (H, T) and (T, H). So, the number of outcomes favourable to E is 3.

Therefore, $P(E) = \frac{3}{4}$

i.e., the probability that Harpreet gets at least one head is $\frac{3}{4}$.

12. The outcomes associated with this experiment are given by

HHH, HHT, HTH, THH, TTH, THT, HTT, TTT

∴ Total number of possible outcomes = 8

Now, Ramesh will lose the game if he gets

HHT, HTH, THH, TTH, THT, HTT

∴ Favourable number of events = 6

∴ Probability that he lose the game $= \frac{6}{8} = \frac{3}{4}$

13. Elementary events associated to random experiment of tossing three coins are

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

∴ Total number of elementary events = 8

(i) The event “getting all heads” is said to occur, if the elementary event HHH occurs, i.e., HHH is an outcome.

∴ Favourable number of elementary events = 1

Hence, required probability $= \frac{1}{8}$

(ii) The event “getting two heads” will occur, if one of the elementary events HHT, THH, HTH occurs.

∴ Favourable number of elementary events = 3

Hence, required probability $= \frac{3}{8}$

(iii) The event of “getting one head”, when three coins are tossed together, occurs if one of the elementary events HTT, THT, TTH, occurs.

Favourable number of elementary events = 3

Hence, required probability $= \frac{3}{8}$

(iv) If any of the elementary events HHH, HHT, HTH, and THH is an outcome, then we say that

the event “getting at least two heads” occurs.

∴ Favourable number of elementary events = 4

honom 4 Hence, required probability $= \frac{4}{8} = \frac{1}{2}$

(v) Similar as (iv) P (getting at least two tails) $= \frac{4}{8} = \frac{1}{2}$

14. We have, the total number of possible outcomes associated with the random experiment of throwing a die is 6 (i.e., 1, 2, 3, 4, 5, 6).

(i) Let E denotes the event of getting a prime number.

So, favourable number of outcomes = 3 (i.e., 2, 3, 5)

$\therefore$ $P(E) = \frac{3}{6} = \frac{1}{2}$

(ii) Let E be the event of getting a number lying between 2 and 6.

$\therefore$ Favourable number of elementary events (outcomes) = 3 (i.e., 3, 4, 5)

$\therefore$ $P(E) = \frac{3}{6} = \frac{1}{2}$

(iii) Let E be the event of getting an odd number.

$\therefore$ Favourable number of elementary events = 3 (i.e., 1, 3, 5)

$\therefore$ $P(E) = \frac{3}{6} = \frac{1}{2}$

15. (i) Here, let E be the event getting a number greater than 4'. The number of possible outcomes are six : 1, 2, 3, 4, 5 and 6, and the outcomes favourable to E are 5 and 6. Therefore, the number of outcomes favourable to E is 2. So,

P(E) = P (number greater than 4) $= \frac{2}{6} = \frac{1}{3}$

(ii) Let F be the event 'getting a number less than or equal to 4'.

Number of possible outcomes = 6

Outcomes favourable to the event F are 1, 2, 3, 4.

So, the number of outcomes favourable to F is 4.

Therefore, $P(F) = \frac{4}{6} = \frac{2}{3}$

Long Answer :

1. Here, total number of possible outcomes = 52

(i) As we know that there are two suits of red card, i.e., diamond and heart and each suit contains one king.

$\therefore$ Favourable number of outcomes = 2

$\therefore$ Probability of getting a king of red colour $= \frac{2}{52} = \frac{1}{26}$

(ii) As we know that kings, queens and jacks are called face cards. Therefore, there are 12 face cards.

∴ Favourable number of elementary events = 12

∴ Probability of getting a face card $= \frac{12}{52} = \frac{3}{13}$

(iii) As we know there are two suits of red cards, i.e., diamond and heart and each suit contains 3 face cards.

∴ Favourable number of elementary events = 2 × 3 = 6

∴ Probability of getting red face card $= \frac{6}{52} = \frac{3}{26}$

(iv) Since, there is only one jack of hearts.

∴ Favourable number of elementary events = 1

∴ Probability of getting the jack of heart $= \frac{1}{52}$

(v) Since, there are 13 cards of spade.

∴ Favourable number of elementary events = 13

∴ Probability of getting a spade $= \frac{13}{52} = \frac{1}{4}$

(vi) Since, there is only one queen of diamonds.

∴ Favourable number of outcomes (elementary events) = 1

∴ Probability of getting a queen of diamond $= \frac{1}{52}$

2. Out of 52 cards, one card can be drawn in 52 ways.

So, total number of elementary events = 52

(i) There are four ace cards in a pack of 52 cards. So, one ace can be chosen in 4 ways.

∴ Favourable number of elementary events = 4

Hence, required probability $= \frac{4}{52} = \frac{1}{13}$

(ii) There are 26 red cards in a pack of 52 cards. Out of 26 red cards, one card can be chosen in 26 ways.

∴ Favourable number of elementary events = 26

Hence, required probability $= \frac{26}{52} = \frac{1}{2}$

(iii) There are 26 red cards, including two red kings, in a pack of 52 playing cards. Also, there are 4 kings, two red and two black. Therefore, card drawn will be a red card or a king if it is any one of 28 cards (26 red cards and 2 black kings).

∴ Favourable number of elementary events = 28

Hence, required probability $= \frac{28}{52} = \frac{7}{13}$

(iv) A card drawn will be red as well as king, if it is a red king. There are 2 red kings in a pack of 52 playing cards.

∴ Favourable number of elementary events = 2

Hence, required probability $= \frac{2}{52} = \frac{1}{26}$

(v) In a deck of 52 cards: kings, queens, and jacks are called face cards. Thus, there are 12 face cards. So, one face card can be chosen in 12 ways.

Favourable number of elementary events = 12

Hence, required probability $= \frac{12}{52} = \frac{3}{13}$

(vi) There are 6 red face cards 3 each from diamonds and hearts. Out of these 6 red face cards, one card can be chosen in 6 ways.

∴ Favourable number of elementary events = 6

Hence, required probability $= \frac{6}{52} = \frac{3}{26}$

(vii) There is only one '2' of spades.

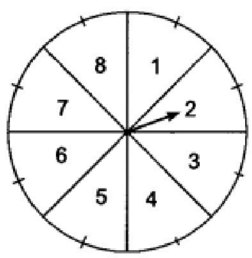
∴ Favourable number of elementary events = 1 Hence, required probability = 2

(viii) There are two suits of black cards viz. spades and clubs. Each suit contains one card bearing number 10.

∴ Favourable number of elementary events = 2

Hence, required probability $= \frac{2}{52} = \frac{1}{26}$

3.



Here, total number of elementary events (possible outcomes) = 8

(i) We have only one 'P' on the spining plant.

∴ Favourable number of outcomes = 1

Hence, the probability that arrow points at 8 $= \frac{1}{26}$.

(ii) We have four odd points (i.e., 1, 3, 5 and 7)

∴ Favourable number of outcomes = 4

∴ Probability that arrow points at an odd number $= \frac{4}{8} = \frac{1}{2}$

(iii) We have 6 numbers greater than 2, i.e., 3, 4, 5, 6, 7 and 8.

Therefore, favourable number of outcomes = 6

∴ Probability that arrow points at a number greater than 2 $= \frac{6}{8} = \frac{3}{4}$

(iv) We have 8 numbers less than 9, i.e, 1, 2, 3, ... 8.

∴ Favourable number of outcomes = 8

∴ Probability that arrow points at a number less than 9 $= \frac{8}{8} = 1$

4. When the blue die shows 'l', the grey die could show any one of the numbers 1, 2, 3, 4, 5, 6.

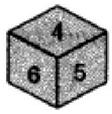
The same is true when the blue die shows '2', '3', '4', '5' or '6'. The possible outcomes of the experiment are listed in the table below; the first number in each ordered pair is the number appearing on the blue die and the second number is that on the grey die. So, the number of possible outcomes = 6 × 6 = 36.

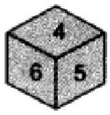
(i) The outcomes favourable to the event the sum of the two numbers is 8' denoted by E, are :

(2, 6), (3, 5), (4, 4), (5, 3), (6, 2) (see figure)

i.e., the number of outcomes favourable to E = 5.

Hence, P(E) $= \frac{5}{36}$





	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

(ii) As you can see from figure, there is no outcome favourable to the event F, 'the sum of two numbers is 13'.

So, $P(F) = \frac{0}{36} = 0$

(iii) As you can see from figure, all the outcomes are favourable to the event G, 'sum of two numbers ≤ 12.

So, $P(G) = \frac{36}{36} = 1.$

5. Total number of cards = 49

Total number of outcomes = 49

(i) Odd number

Favourable outcomes : 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49

Number of favourable outcomes = 25

$$\text{Probability } (E) = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{25}{49}$$

(*ii*) A multiple of 5
Favourable outcomes: 5, 10, 15, 20, 25, 30, 35, 40, 45

Number of favourable outcomes $= 9$

$$\text{Probability } (E) = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{9}{49}$$

(*iii*) A perfect square

Favourable outcomes: 1, 4, 9, 16, 25, 36, 49

Number of favourable outcomes $= 7$

$$\text{Probability } (E) = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{7}{49} = \frac{1}{7}$$

(*iv*) An even prime number

Favourable outcome $= 2$

Number of favourable outcome $= 1$

$$\text{Probability } (E) = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{1}{49}$$

Case Study Answer-

1. Answer:

Total number of days in June = 30

i. (c) 15

Solution:

Number of sunny days = P(sunny day) × 30 $= \frac{1}{2} \times 30 = 15$

ii. (b) $\frac{1}{6}$

Solution:

Number of cloudy days in June = 5

$$\therefore x = \frac{5}{30} = \frac{1}{6}$$

iii. (a) $\frac{13}{15}$

Solution:

Required probalility $= \frac{1}{2} + \frac{1}{6} + \frac{1}{5} = \frac{13}{15}$

iv. (d) 4

Solution:

We have, $x + y = \frac{3}{10} \Rightarrow y = \frac{3}{10} - \frac{1}{6} = \frac{2}{15}$

So, number of rainy days $= \frac{2}{15} \times 30 = 4$

v. (c) 6

Solution:

Number of partially cloudy days = p(partially cloudy days) × 30 $= \frac{1}{5} \times 30 = 6$

2. Answer:

Total number of cartons in the store = 80 + 90 + 38 + 42 = 250

i. (d) $\frac{9}{25}$

Solution:

P(choosing an apple juice carton) $= \frac{90}{250} = \frac{9}{25}$

ii. (c) $\frac{17}{25}$

Solution:

P(choosinganorangejuicecarton) $= \frac{80}{250} = \frac{8}{25}$

$\therefore$ p(choosing not an orange juice carton) $= 1 - \frac{8}{25} = \frac{17}{25}$

iii. (d) $\frac{21}{125}$

Solution:

p(choosing a guava juice carton) $= \frac{42}{250} = \frac{21}{125}$

iv. (c) $\frac{3}{10}$

Solution:

Total number of cartons Vishal bought =4 + 3 + = 10

Number of tetrapacks in I carton = 10

$\therefore$ Total number of tetrapacks Vishal has = 100

So, p(customer picks a guava juice tetrapack) $= \frac{3\times10}{100} = \frac{3}{10}$

v. (b) $\frac{50}{127}$

Solution:

Number of cartons left with storekeeper = 250 - 10 = 240

Number of cartons he bought = 14

$\therefore$ Total number of cartons are with storekeeper now = 240 + 14 = 254

So, p(selecting a tetrapack of apple juice from store),

$$= \frac{(90-4+14)\times10}{254\times10} = \frac{100}{254} = \frac{50}{127}$$

Assertion Reason Answer-

1. (a) Both A and R are true and R is the correct explanation of A.
2. (a) Both A and R are true and R is the correct explanation of A.